5.5 Linear model

In this section an attempt is made to parametrize the response of LUCID ($N$) with a linear function of $\mu$ through the origin:

$$N(\mu) = k_{\text{LUCID}} \times \mu$$

The calibration constant ($k_{\text{LUCID}}$) is extracted from the point at $\mu = 0.01$.

5.5.1 Results with zero counting

The linear model with zero counting is tested using the opposite of the logarithm of the fraction of empty events:

$$- \log(N_{0/BX}) = k_{\text{LUCID}} \times \mu$$

In Figure 25, the measurements at different $\mu$ are compared with the prediction of the linear model, for four values of the threshold (40, 50, 60, 70 p.e.).

**Figure 25.** Comparison between measured LUCID response (dots) and prediction of the linear model (solid line) with zero counting method in single side (top-left) and coincidence mode (bottom-left). The deviation from the prediction is shown in the right plots.

In single side mode, with a threshold of 50 p.e., the calibration constant is $k_{\text{LUCID}} = 0.551 \pm 0.005$ and the deviation from linearity is less than 2% when
In single side mode, for $\mu < 2$, the agreement between the measured and the expected number of interactions is within $1\%$, when a threshold of 50 p.e. is set. At $\mu = 5$, migration effect starts to play a role. The probability to detect an interaction increases with $\mu$ compared to the calibration scenario. The number of observed empty bunch crossings is smaller than the one measured with an ideal detector without migration effect, causing an overestimate of $\mu$.

In coincidence mode, for $\mu < 1$, the agreement between the measured and the expected number of interactions is within $2\%$, when a threshold of 50 p.e. is set. At $\mu = 2$, migration effect starts to play a role. The probability to detect an interaction increases with $\mu$ compared to the calibration scenario. The number of empty crossings is smaller than the expected value causing an overestimate of $\mu$.

In both detection modes, for $\mu > 5$ the statistical uncertainty becomes dominant due to lack of empty events.
5.5 Linear model

$\mu < 2$. In coincidence mode, with the same threshold, $k_{LUCID} = 0.135 \pm 0.002$ and the deviation from linearity is already 4% at $\mu = 0.1$. The deviation from linearity is due to the migration effect. The probability to detect an interaction increases with $\mu$, therefore the number of empty bunches is smaller than the expected one, causing an overestimate of $\mu$. It can be noticed that the calibration constant in single side mode is equal to the detection efficiency ($\epsilon^{Sing}$, see appendix A).

5.5.2 Results with hit counting

The linear model with hit counting is tested using the number of hits per bunch:

$$N_{hits/BX} = k_{LUCID} \times \mu$$

In Figure 26, the number of hits per bunch at different $\mu$ is compared with the prediction of the linear model, for four values of the threshold (40, 50, 60, 70 p.e.).

![Figure 26](image)

*Figure 26. Comparison between measured LUCID response (dots) and prediction of the linear model (solid line) with hit counting method in single side (top-left) and coincidence mode (bottom-left). The deviation from the prediction is shown in the right plots.*

In single side mode, with a threshold of 50 p.e., the calibration constant is $k_{LUCID} = 1.120 \pm 0.014$ and the deviation from linearity is 5% when $\mu < 2$. In coincidence mode, with the same threshold, $k_{LUCID} = 0.489 \pm 0.010$ and the deviation from linearity is already 5% at $\mu = 0.1$.  


5.6 Combinatorial model

Figure 28. Comparison between the average number of interactions per bunch crossing measured with LUCID (dots) and predicted with the combinatorial model (solid line) by counting the number of hits in single side (top-left) and coincidence mode (bottom-left). The deviation from the prediction is shown in the right plots.

In single side mode, for \( \mu < 1 \), the agreement between the measured and the expected number of interactions is within 2.6%, when a threshold of 50 p.e. is set. For larger values of \( \mu \), the disagreement increases exponentially due to migration effect which causes an overestimate of \( \mu \).

In coincidence mode, already at \( \mu = 1 \), the disagreement between the measured and the expected number of interactions is 6%, when a threshold of 50 p.e. is set. For larger values of \( \mu \), the disagreement increases exponentially due to migration effect which causes an overestimate of \( \mu \).
5.7 Polynomial model

Figure 29. Polynomial fit of $N_{\text{hits}}/B_X$ as a function of the true value in single side (top-left) and coincidence mode (bottom-left). The deviation from the fit is shown in the right plots.

Figure 30. Comparison between the average number of interactions per bunch crossing measured with LUCID (dots) and predicted with the polynomial model (solid line) by counting the number of hits in single side (top-left) and coincidence mode (bottom-left). The deviation from the prediction is shown in the right plots.