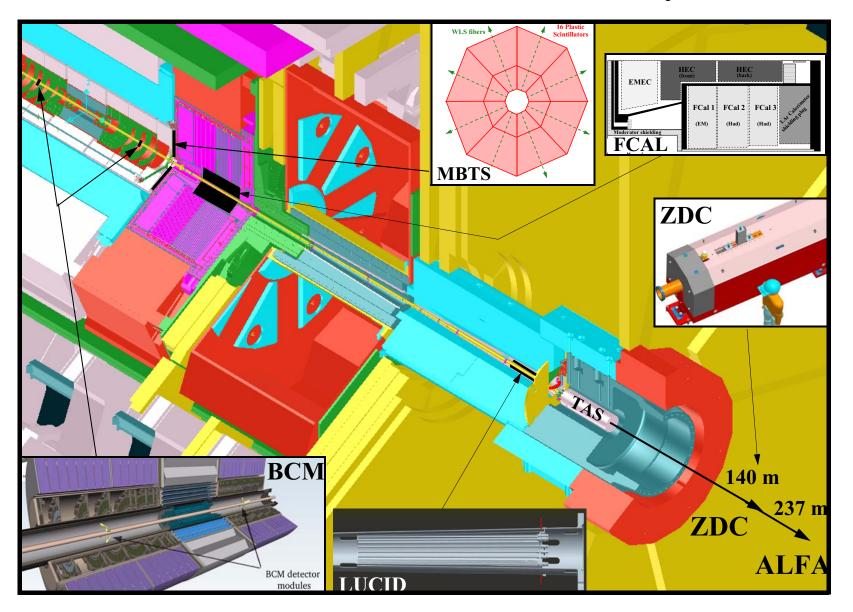
Report from the luminosity measurement taskforce



http://project-atlas-lucid.web.cern.ch/project-atlas-lucid/taskforce/main.html



The basic principle



The luminosity (L) can be calculated from the rate of all inelastic interactions (R_{in}) and the total inelastic cross section (σ_{in}) by using the simple relation:

$$L = \frac{R_{in}}{\sigma_{in}}$$

The inelastic interaction rate is given by the average number of inelastic interactions per bunch crossing (μ) and the bunch crossing rate (f_{BX}):

$$R_{in} = \mu \times f_{BX} = \mu \times \frac{\text{The Number of filled Bunch crossings}}{3564} \times 40 \text{ MHz}$$

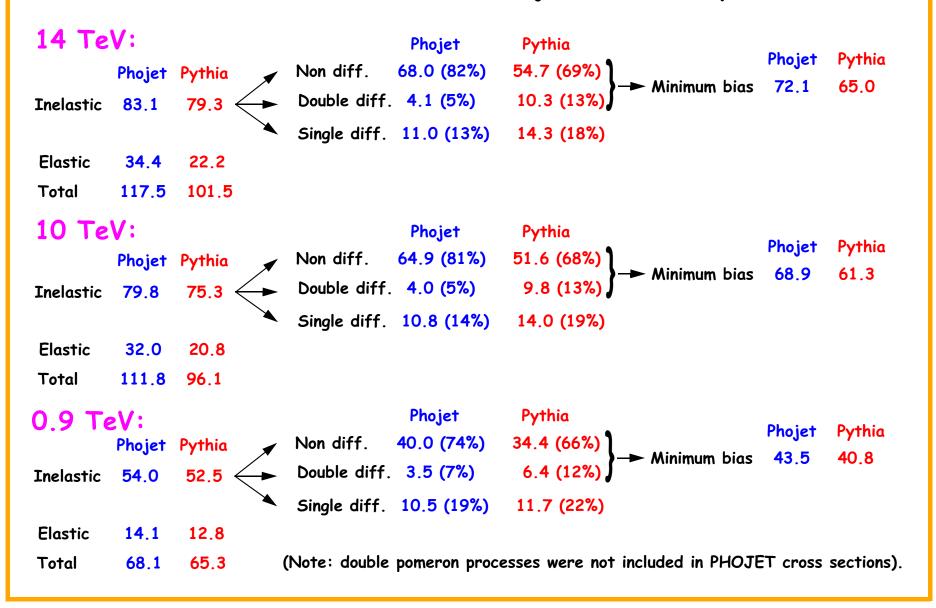
Conclusion: To provide a luminosity the detector must estimate μ correctly from a measured rate and the inelastic cross section has to be known.



The inelastic cross section



Predicted cross section in mb from Phojet 1.12 and Pythia 6.420





Measuring μ

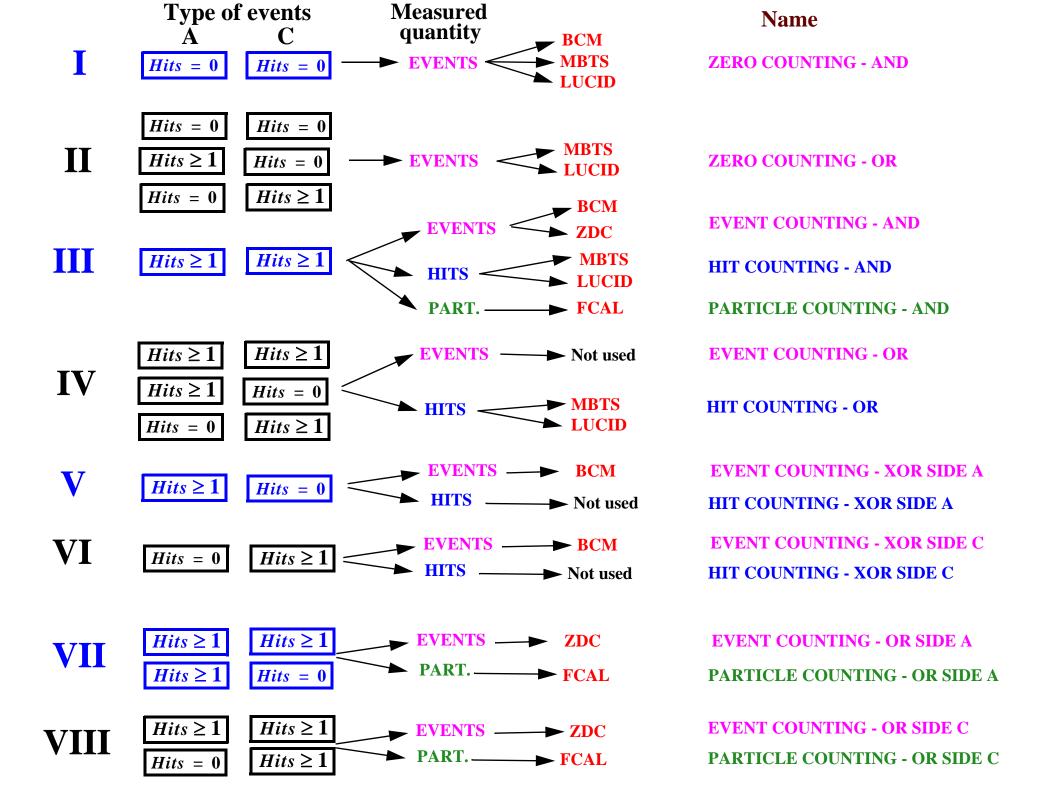


The average number of interactions per bunch crossing (μ) is estimated from a rate measured by the detectors.

The measured rate that is used to estimate μ can be of three main types:

- 1) Event rate
- 2) Hit rate
- 3) Particle rate

The different detectors measure different quantities while requiring different combinations of signals in the two detectors. This lead to a long list of different luminosity methods.





Measuring μ by particle counting



If a true number of particles is counted in the detectors without any requirement on a minimum number of particles then

$$\mu = \frac{N_{part/BX}}{N_{part/pp}} = \frac{\text{The average number of detected particles per bunch crossing}}{\text{The average number of detected particles per inelastic pp interaction}}$$

The average number of detected particles per inelastic pp interaction ($N_{part/pp}$) has to be obtained from simulations or from a reference data sample recorded at low luminosity (low μ).

Another possibility is to measure the luminosity with for example ALFA and calibrate the detector:

$$L = \frac{f_{BX}}{\sigma_{in}} \times \frac{N_{part/BX}}{N_{part/pp}}$$

$$\sigma_{in} \times N_{part/pp} = \frac{L}{N_{part/BX}} \times \frac{1}{f_{BX}}$$
Detector

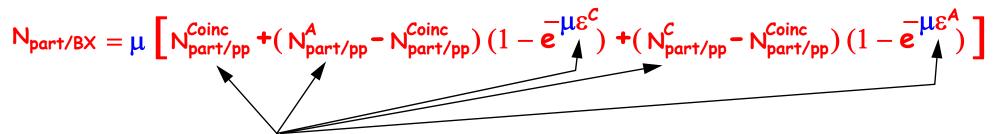


Particle counting with coincidence



In order to reduce background it is an advantage to do particle counting while requiring at least one particle in each detector. In this case μ is no longer proportional to $N_{part/BX}$.

If only events with particles in both detectors are used then the relationship between $N_{\text{part/bx}}$ and μ becomes:



5 parameters needed to describe the dependence on μ !

What is needed is μ as a function of $N_{part/BX}$ i.e. μ = f(Npart/BX) but the expression above cannot be inverted analytically.



Measuring μ by hit counting



Most detectors do not measure particles. If hits are measured instead without any requirement on a minimum number of hits in each detector there is no longer a linear relationship between μ and $N_{hits/BX}$:

$$\begin{split} N_{\text{hits/BX}} &= N_{\text{tubes}} \left[1 - \left(1 - \frac{N_{\text{hits/pp}}}{N_{\text{tubes}}} \right)^{\mu} \right] \approx \mu \ N_{\text{hits/pp}} \quad \text{for } \mu << 1 \\ \mu &= \frac{\ln \left(1 - \frac{N_{\text{hits/BX}}}{N_{\text{tubes}}} \right)}{\ln \left(1 - \frac{N_{\text{hits/pp}}}{N_{\text{tubes}}} \right)} \end{split}$$

$$L = \frac{f_{\text{BX}}}{\sigma_{\text{in}}} \times \frac{\ln \left(1 - \frac{N_{\text{hits/BX}}}{N_{\text{tubes}}} \right)}{\ln \left(1 - \frac{N_{\text{hits/pp}}}{N_{\text{tubes}}} \right)}$$

 N_{tubes} is here the number of active detector elements (30 for LUCID and 32 for MBTS) and $N_{hits/pp}$ is the average number of hits per bunch crossing when there is exactly one interaction in the events (1.2 for LUCID at 14 TeV).



Hit counting with coincidence



In order to reduce background it is an advantage to do also hit counting while requiring at least one particle in each detector.

The relationship between the number of hits and particles:

$$N_{part} = -N_{tubes} \times \ln \left(1 - \frac{N_{hits}}{N_{tubes}}\right)$$

can be used together with the relationship between μ and the number of particles

$$N_{part/BX} = 2\mu N_{part/pp}^{A} (1 - e^{-\mu\epsilon^{A}}) - \mu N_{part/pp}^{Coinc} (1 - 2e^{-\mu\epsilon^{A}}) \qquad \text{if} \quad N_{part/pp}^{A} = N_{part/pp}^{Coinc} \text{ and } \epsilon^{A} = \epsilon^{C} = N_{part/pp}^{A} = N_{part/pp}^{C} = N_{pa$$

to obtain a relationship between N_{hits} and μ :

Nhits = N_{tubes}
$$\left[1 - e^{-(2\mu N_{part/pp}^{A}(1 - e^{-\mu \epsilon^{A}}) - \mu N_{part/pp}^{Coinc}(1 - 2e^{-\mu \epsilon^{A}}))/N_{tubes}}\right]$$

However, there is no way of analytically express μ as a function of N_{hits} in this case !

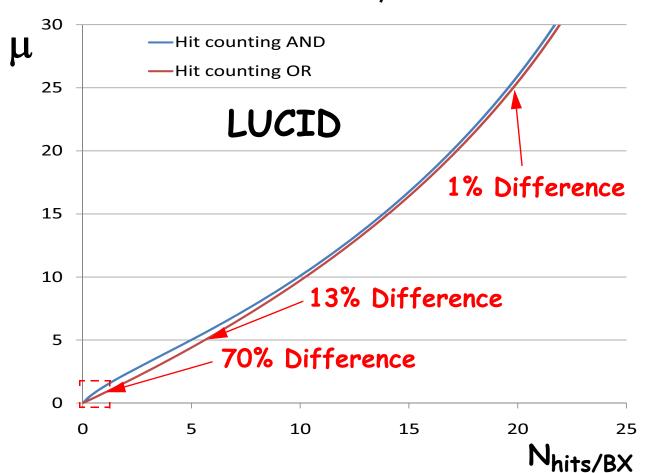


Hit counting - AND versus OR

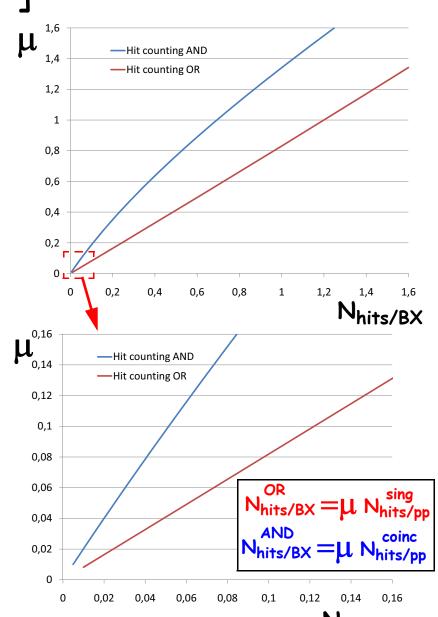


$$N_{\text{hits}}^{\text{AND}} = N_{\text{tubes}} \left[1 - e^{-(2\mu N_{\text{part/pp}}^{\text{A}})(1 - e^{-\mu \epsilon^{\text{A}}}) - \mu N_{\text{part/pp}}^{\text{Coinc}}(1 - 2e^{-\mu \epsilon^{\text{A}}}) \right] / N_{\text{tubes}}$$

$$N_{\text{hits/BX}}^{OR} = N_{\text{tubes}} [1 - (1 - N_{\text{hits/pp}} / N_{\text{tubes}})^{\mu}]$$



The dependence of μ on N_{hits} is only linear for small $\mu.$ The largest difference between AND and OR is at low $\mu.$

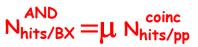


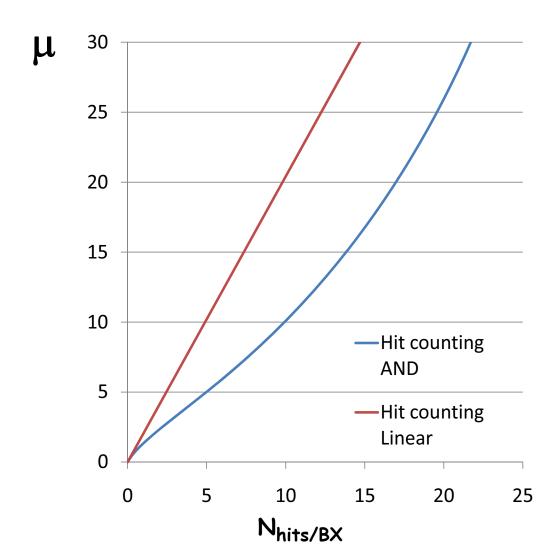


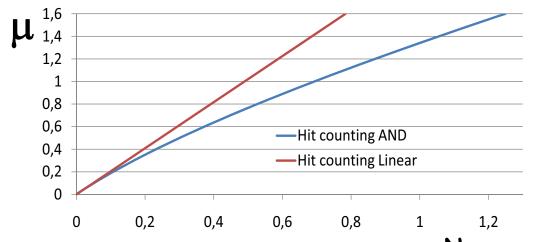
Hit counting - linear extrapolation

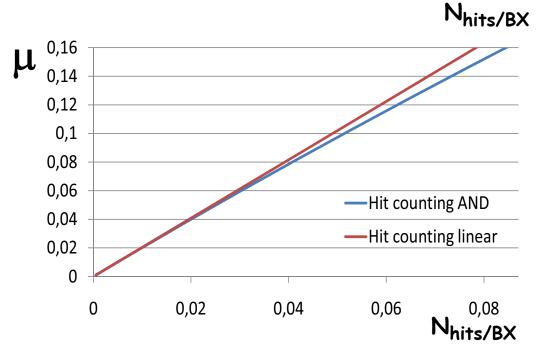


$$N_{\text{hits}}^{\text{AND}} = N_{\text{tubes}} \left[1 - e^{-(2\mu N_{\text{part/pp}}^{\text{A}} (1 - e^{-\mu \epsilon^{\text{A}}}) - \mu N_{\text{part/pp}}^{\text{Coinc}} (1 - 2e^{-\mu \epsilon^{\text{A}}}))/N_{\text{tubes}}} \right]$$











Event counting



The simplest quantity to measure is the number of events per bunch crossing.

Different requirements can be made on the signals seen in the two detectors and different system uses different requirement.

Some of these event topologies are not independent with respect to each other.

```
Probability( Zero-counting-AND ) = 1 - Probability( Event-counting-OR)
Probability( Zero-counting-OR ) = 1 - Probability( Event-counting-AND)
```

It is possible to calculate the probability (rate) of any event topology at any μ if three basic parameters (ϵ_0 , ϵ_1 and ϵ_2), that describe the probability when there is one pp interaction, are known.

A problem with event counting is that at high μ the event rate tends to saturate and an increase in μ does then not give a significant increase in the event rate.



Event counting



<u>Name</u>	Event type	Propability for 1 pp	Probability for μ interactions
ZERO COUNTING - AND	Hits = 0 $Hits = 0$	$\varepsilon_0 = 1 - \varepsilon_1 - \varepsilon_2 - \varepsilon_3$	$e^{(\epsilon_0-1)\mu}$
EVENT COUNTING - XOR - A	$Hits \ge 1$ $Hits = 0$	$\varepsilon_1 = \varepsilon_A - \varepsilon_{coinc}$	$e^{(\epsilon_0+\epsilon_1-1)\mu}-e^{(\epsilon_0-1)\mu}$
EVENT COUNTING - XOR - C	$Hits = 0$ $Hits \ge 1$	$\varepsilon_2 = \varepsilon_C - \varepsilon_{coinc}$	$e^{(\epsilon_0+\epsilon_2-1)\mu}$ - $e^{(\epsilon_0-1)\mu}$
EVENT COUNTING - AND	$Hits \ge 1$ $Hits \ge 1$	$\varepsilon_3 = \varepsilon_{\text{coinc}}$	1 - $e^{(\epsilon_0 + \epsilon_1 - 1)\mu}$ - $e^{(\epsilon_0 + \epsilon_2 - 1)\mu}$ + $e^{(\epsilon_0 - 1)\mu}$
EVENT COUNTING - OR	$Hits \ge 1$ $Hits \ge 1$ $Hits = 0$ $Hits \ge 1$	$\varepsilon_{\rm sing} = 1 - \varepsilon_0$	$1 - e^{(\epsilon_0 - 1)\mu}$
ZERO COUNTING - OR	$Hits = 0$ $Hits = 0$ $Hits \ge 1$ $Hits = 0$ $Hits \ge 1$	-	$e^{(\epsilon_0 + \epsilon_1 - 1)\mu} + e^{(\epsilon_0 + \epsilon_2 - 1)\mu} - e^{(\epsilon_0 - 1)\mu}$
EVENT COUNTING - OR - A	$Hits \ge 1$ $Hits \ge 1$ $Hits \ge 0$	$ \epsilon_{\mathbf{A}} = 1 - \epsilon_0 - \epsilon_2 $	1 - $e^{(\epsilon_0 + \epsilon_2 - 1)\mu}$
EVENT COUNTING - OR - C	$Hits \ge 1$ $Hits \ge 1$ $Hits \ge 1$	$\varepsilon_{\rm C} = 1 - \varepsilon_0 - \varepsilon_1$	$1 - e^{(\epsilon_0 + \epsilon_1 - 1)\mu}$



Event counting



Efficiencies for one pp interaction at 14 TeV

A	C	LUCID	BCM	MBTS	ZDC	$\varepsilon_{\rm sing} = 1 - \varepsilon_0$	LUCID	BCM	MBTS	ZDC
						$\varepsilon_{\mathbf{A}} = 1 - \varepsilon_0 - \varepsilon$				
Hits = 0	$Hits \ge 1$	$\mathcal{E}_2 = 0.212$	0.125	0.004	0.215	$\varepsilon_{\mathbf{C}} = 1 - \varepsilon_0 - \varepsilon$	$s_1 = 0.347$	0.165	0.998	0.313
$Hits \ge 1$	$Hits \ge 1$	$\mathbf{E}_3 = 0.135$	0.040	0.992	0.098	$\varepsilon_{\text{coinc}} = 1 - \varepsilon_0 - \varepsilon_1 - \varepsilon_0$	$\varepsilon_2 = 0.135$	0.040	0.992	0.098

LUCID: GEANT3 + PHOJET (non-diff + diff, 16 + 16 tubes, cut = 50 p.e.)

BCM: GEANT4 + PYTHIA (non-diff + diff, 4 + 4 modules)

MBTS: GEANT4 + PYTHIA (only non-diff, 16 + 16 sectors, cut = 40 mV)

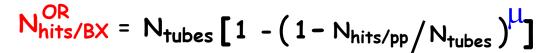
ZDC: ATL-LUM-INT-2009-002, $\varepsilon_A = \varepsilon_C = \sqrt{\varepsilon_{coinc}}$

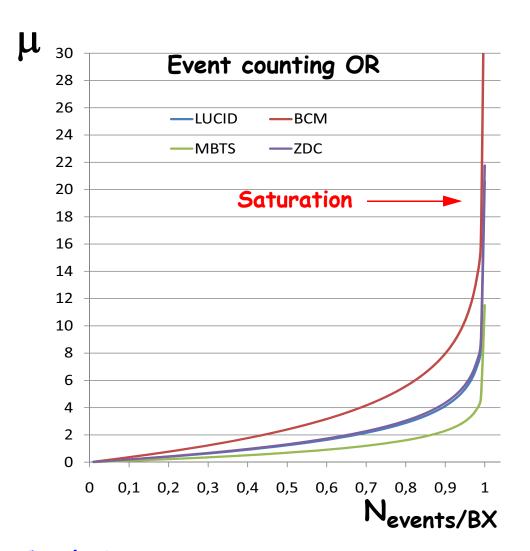


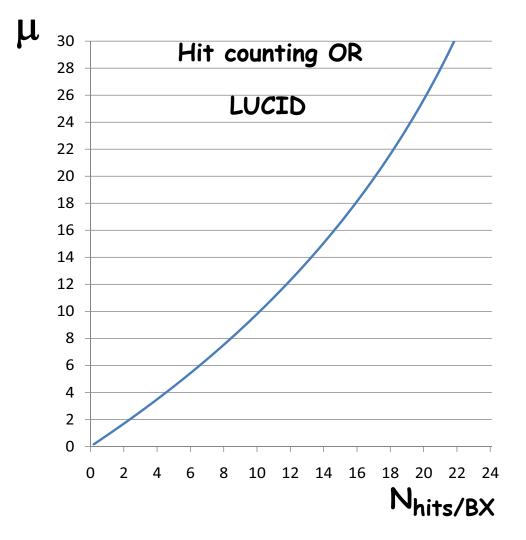
Event versus hit counting



$$N_{\text{events/BX}}^{OR} = 1 - e^{-\epsilon_{\text{sing}}\mu}$$







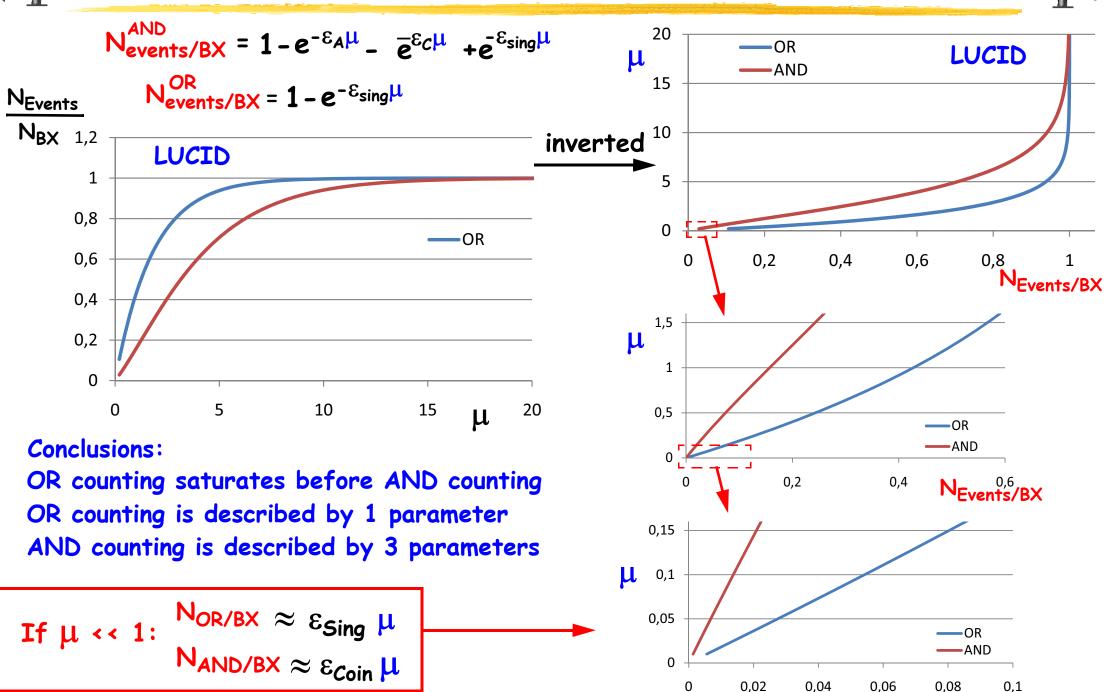
Conclusions:

Hit counting suffers less from saturation than event counting. A small acceptance gives less saturation (but less statistics).



Event counting - AND versus OR





N_{Events/BX}



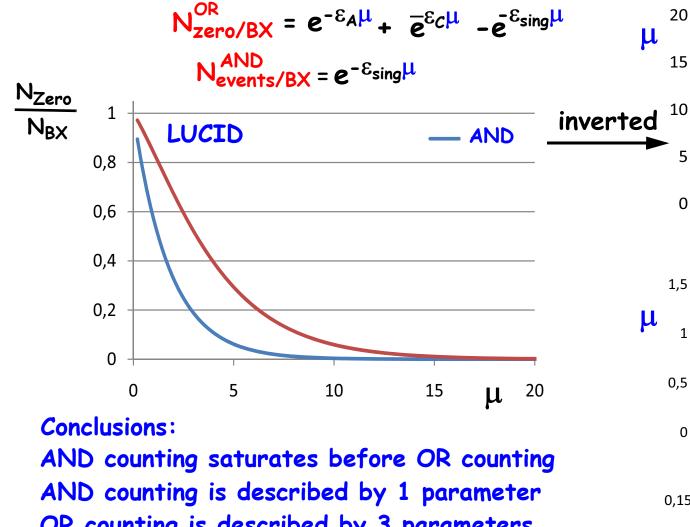
Zero counting - AND versus OR

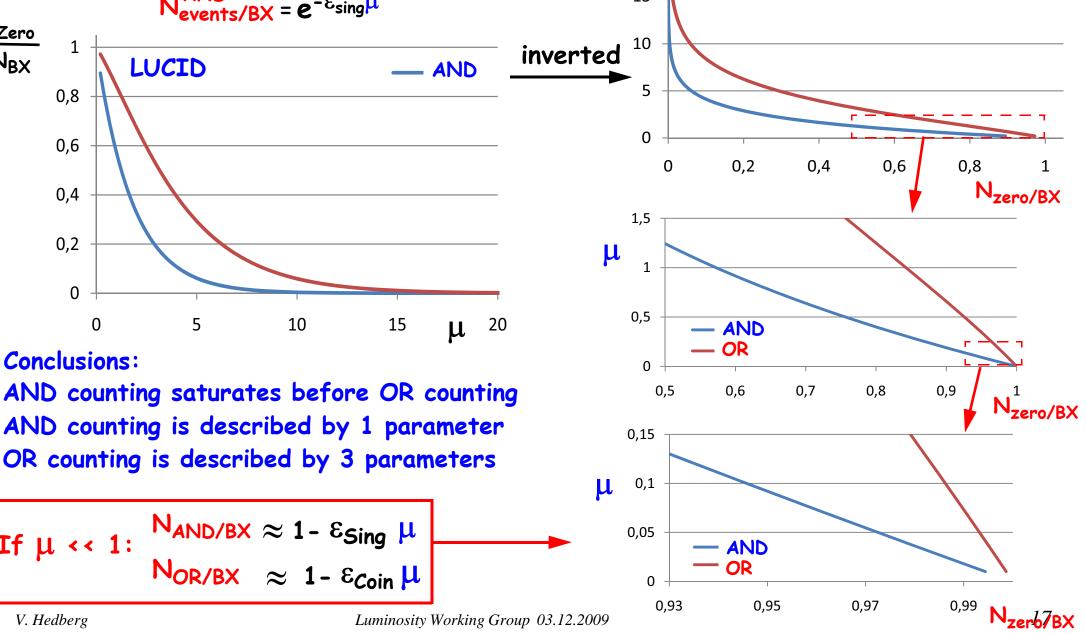


AND

OR

LUCID





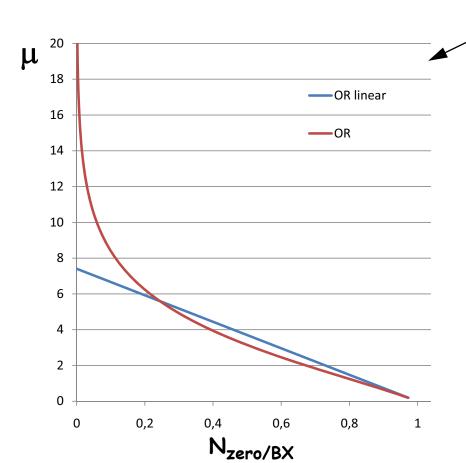
If $\mu \leftrightarrow 1$:



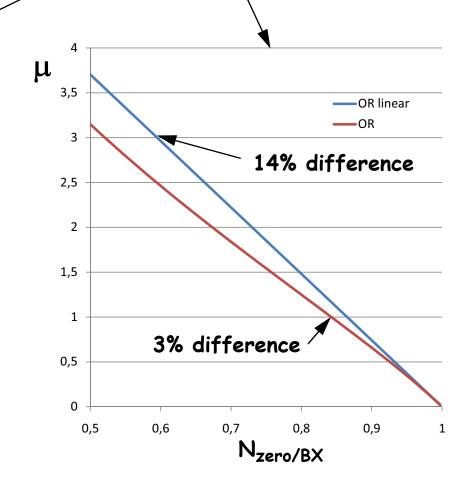
Zero counting - linear extrapolation



Compare Zero-OR counting with a linear extrapolation from the low μ limit:



 $N_{\text{zero/BX}}^{\text{OR}} = e^{-\epsilon_A \mu} + \bar{e}^{\epsilon_C \mu} - \bar{e}^{\epsilon_{\text{sing}} \mu}$ $N_{\text{zero/BX}}^{\text{OR}} = 1 - \epsilon_{\text{Coin}} \mu$

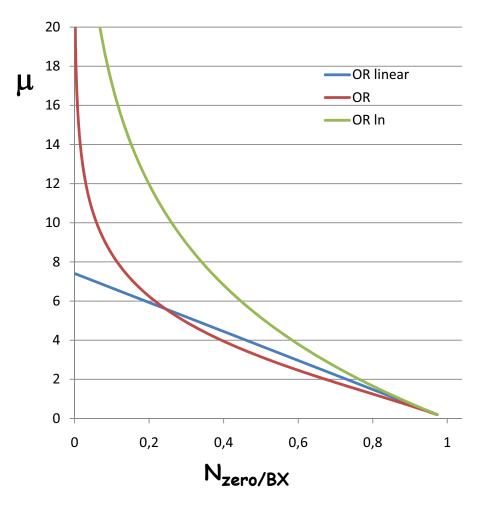


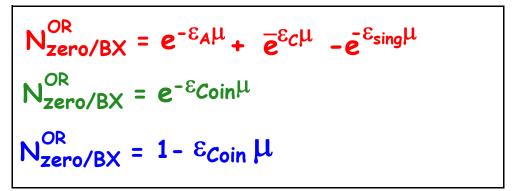


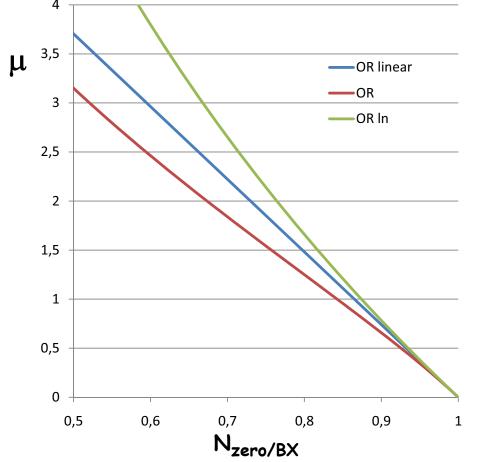
Zero counting - more extrapolation



Compare Zero-OR counting with a linear extrapolation from the low μ limit and an exponential extrapolation:



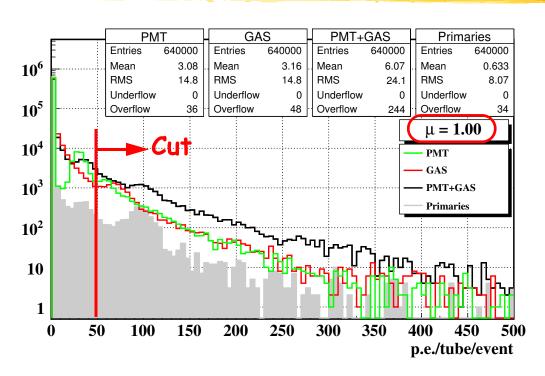


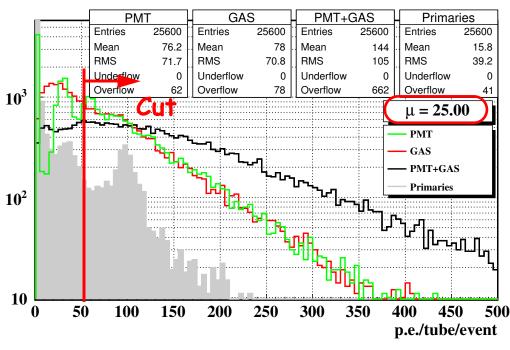




Migration



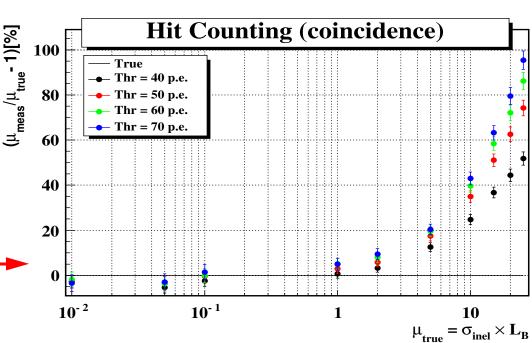




Migration:

When events are piled up at large μ , particles that give a small signal combines with other small signals to give a hit above the threshold.

The effect in LUCID is large





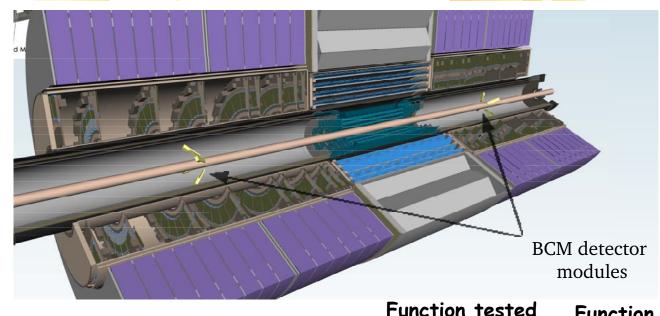
The Beam Condition Monitor



The BCM has four modules on each side.

2+2 modules are at present read out separately from the other 2+2 modules.

The detector will do event counting and not hit counting. Since the detector read-out is split in two one can make two independent measurements for each methods.



Zero-counting-AND:

$$\mu = \frac{\ln\left(\frac{1}{N_{BX}}\right)}{\epsilon_0 - 1}$$

YES

NO

Function

inverted:

Event-counting-AND:

$$\mu = \frac{\ln\left(\frac{N_{00}}{N_{BX}}\right)}{\varepsilon_{0}-1} - \varepsilon_{C} - \varepsilon_{A} - \varepsilon_{sing}$$

$$\frac{N_{AND}}{N_{DX}} = 1 - e^{(\varepsilon_{0}+\varepsilon_{1}-1)\mu} - e^{(\varepsilon_{0}+\varepsilon_{2}-1)\mu} + e^{(\varepsilon_{0}-1)\mu}$$

YES

Event-counting-XOR Side A:

$$\frac{N_A}{N_{BX}} = e^{(\epsilon_0 + \epsilon_1 - 1)\mu} - e^{(\epsilon_0 - 1)\mu}$$

NO

with MC data:

YES

NO

Event-counting-XOR Side C:

$$\frac{N_c}{N_{PY}} = e^{(\epsilon_0 + \epsilon_2 - 1)\mu} - e^{(\epsilon_0 - 1)\mu}$$

NO

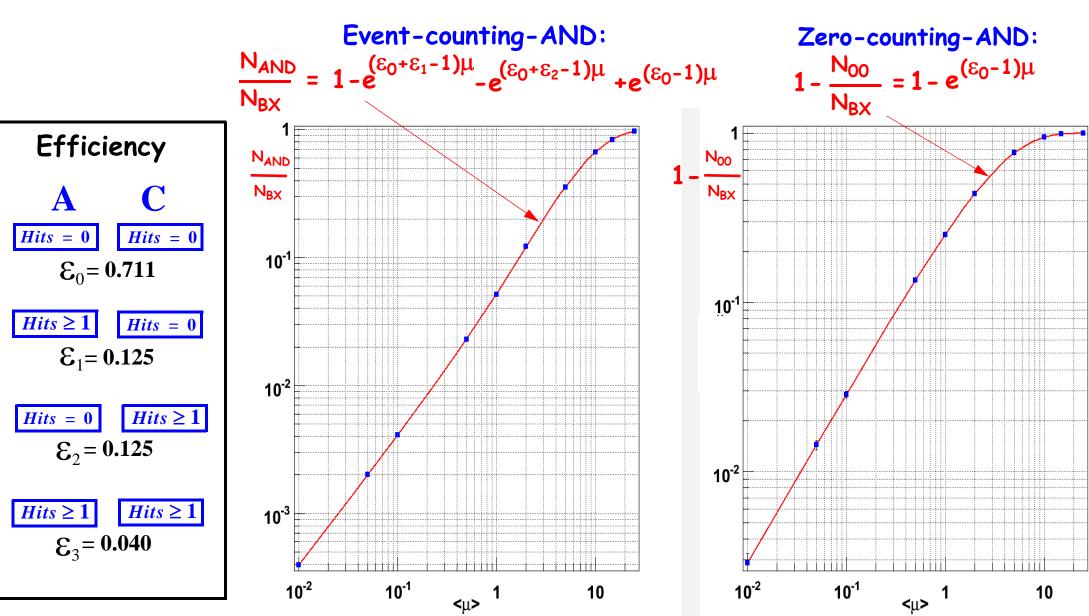
NO



The Beam Condition Monitor



The BCM has so far only been studied with 14 TeV PYTHIA/GEANT4 events and assuming a read-out of 4 modules on each side.



Luminosity Working Group 03.12.2009



The Beam Condition Monitor



What needs to be done?

The Event-Counting-AND formula needs to be inverted.

The Event-Counting-XOR formula needs to be tested with simulated events and inverted (perhaps inclusive OR would be easier?)

The efficiencies needs to be calculated at lower energies than 14 TeV.



The Zero Degree Calorimeter

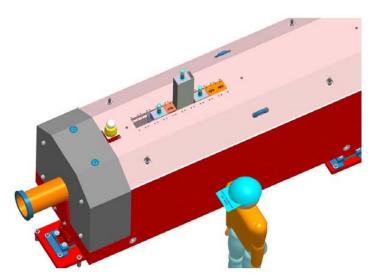


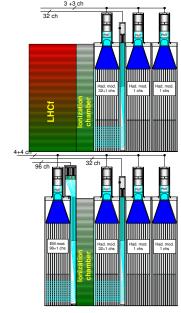
The ZDC has two calorimeters that will measure the total energy. A cut will be made on this energy and three rates will be measured:

ZDC12Rate: Inclusive single rate on Side 12

ZDC81Rate: Inclusive single rate on Side 81

ZDCcoincRate: Coincidence rate





The methods that will be used are:

Event-counting-AND:
$$\frac{N_{AND}}{N_{BX}} = 1 - e^{(\epsilon_0 + \epsilon_1 - 1)\mu} - e^{(\epsilon_0 + \epsilon_2 - 1)\mu} + e^{(\epsilon_0 - 1)\mu} = \frac{ZDC_{coincRate}}{BX \text{ rate}}$$

Event-counting-OR Side A:
$$\frac{N_A}{N_{BX}} = 1 - e^{(\epsilon_0 + \epsilon_2 - 1)\mu} = \frac{ZDC12Rate}{BX \text{ rate}}$$

Event-counting-OR Side C:
$$\frac{N_C}{N_{BX}} = 1 - e^{(\epsilon_0 + \epsilon_1 - 1)\mu} = \frac{ZDC81Rate}{BX \text{ rate}}$$

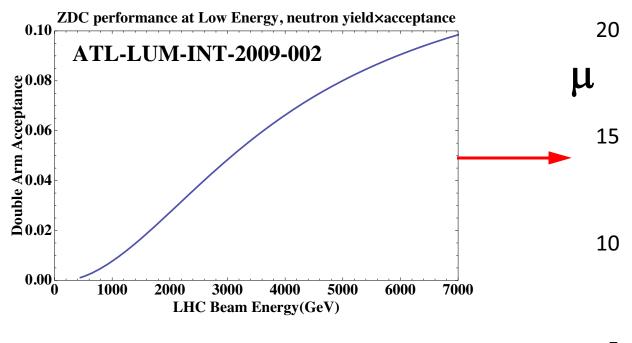
$$\mu = \frac{\ln(1 - \frac{\text{ZDC12Rate}}{\text{BX rate}})}{\epsilon_0 + \epsilon_2 - 1}$$

$$\mu = \frac{\ln(1 - \frac{\text{ZDC81Rate}}{\text{BX rate}})}{\epsilon_0 + \epsilon_1 - 1}$$



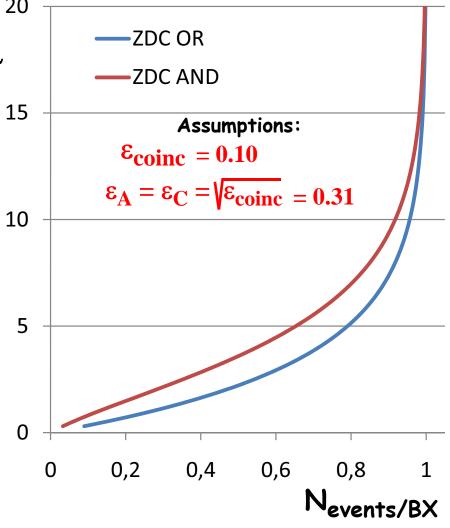
The Zero Degree Calorimeter





What about migration? Accidentals?

A simulation is needed!





The Zero Degree Calorimeter



What needs to be done?

A simulation of the detector is badly needed.

The probability functions have to be tested with simulated data.

Functions for μ = f(ZDC rate) have to be constructed at various energies.

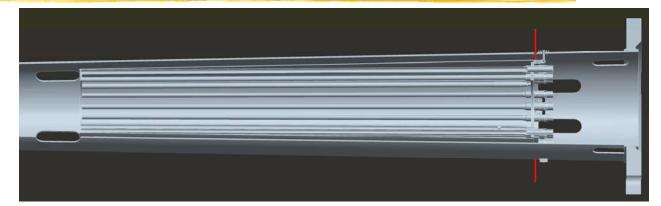


LUCID



LUCID has 15 working Cherenkov tubes in each detector.

The signals are discriminated and provide a hit pattern that are used to do four measurements.



Zero-counting-AND, Zero-counting-OR, Hit-counting-AND, Hit-counting-OR.

Monte Carlo studies have shown that none of the probability functions describe the Monte Carlo data sufficiently well. The reason for this is migration which cannot be described in these functions.

LUCID plots μ as a function of the probability (P) of events or hits and fit a polynomial function so that μ can be expressed as

$$\mu = p_0 + p_1 P + p_2 P^2 + p_3 P^3$$
 where $P = \frac{N_{00}}{N_{BX}} \text{ or } \frac{N_0}{N_{BX}} \text{ or } \frac{N_{hits}}{N_{BX}}$ and p_0 , p_1 , p_3 are constants.

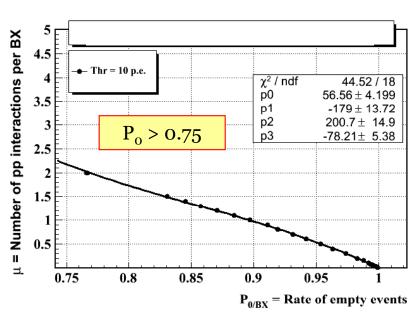
Sometimes the fit is done in two regions of P in order to get a good agreement between the fit and the Monte Carlo data.

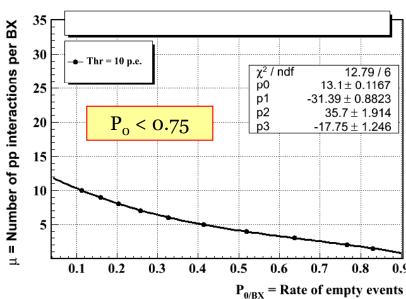


LUCID 900 GeV

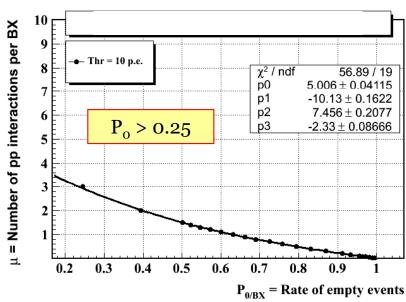


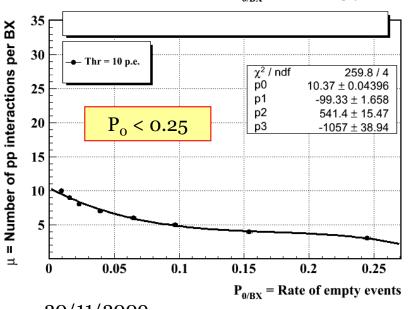
Zero counting (OR)





Zero counting (AND)



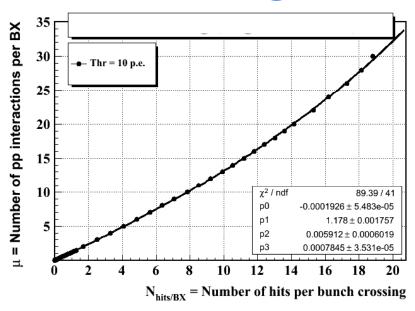




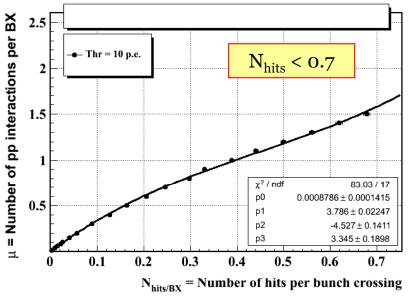
LUCID 900 GeV

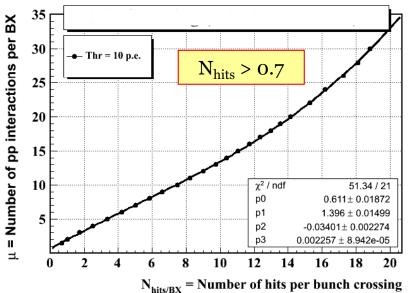


Hit counting (OR)



Hit counting (AND)







Low μ approximation for LUCID at 0.9 TeV



$$N_{\text{zero/BX}}^{OR} = e^{-\epsilon_A \mu} + \overline{e}^{\epsilon_C \mu} - e^{-\epsilon_{\text{sing}} \mu} = e^{-\epsilon_{\text{coin}} \mu} = 1 - \epsilon_{\text{coin}} \mu$$
 $N_{\text{events/BX}}^{AND} = e^{-\epsilon_{\text{sing}} \mu} = 1 - \epsilon_{\text{sing}} \mu$

$$\mu = (1 - N_{\text{zero/BX}}^{OR}) / \epsilon_{\text{coin}}$$

$$\mu = (1 - N_{\text{events/BX}}^{AND}) / \epsilon_{\text{sing}}$$

Threshold
$$\epsilon_A$$
 ϵ_C ϵ_{sing} ϵ_{coin} 10 pe 0.269 0.273 0.463 0.0788 15 pe 0.239 0.243 0.420 0.0625 50 pe 0.104 0.107 0.199 0.0124

$$\begin{split} N_{\text{hits}}^{\text{AND}} &= N_{\text{tubes}} \Big[1 - e^{-(2\mu \ N_{\text{part/pp}}^{\text{A}} (1 - e^{-\mu \epsilon^{\text{A}}}) - \mu \ N_{\text{part/pp}}^{\text{Coinc}} (1 - 2e^{-\mu \epsilon^{\text{A}}})) / N_{\text{tubes}}} \Big] = \mu \ N_{\text{hits/pp}}^{\text{coinc}} \\ N_{\text{hits/BX}}^{\text{OR}} &= N_{\text{tubes}} \Big[1 - \Big(1 - N_{\text{hits/pp}} / N_{\text{tubes}} \Big) \ \Big] = \mu \ N_{\text{hits/pp}}^{\text{sing}} / pp \end{split}$$

$$\mu = N_{hits/BX}^{AND} / N_{hits/pp}^{coinc}$$

$$\mu = N_{hits/BX}^{OR} / N_{hits/pp}^{sing}$$

Threshold	NAhits/pp	$N^{\mathcal{C}}_{hits/pp}$	Nsing Nhits/pp	N ^{coinc} hits/pp
10 pe	0.550	0.559	0.856	0.2530
15 pe	0.450	0.458	0.719	0.1890
50 pe	0.138	0.142	0.250	0.0299



Measured μ for LUCID at 0.9 TeV



Selection of MBTS events using pulseheight and timing cuts and requiring hits on both sides.

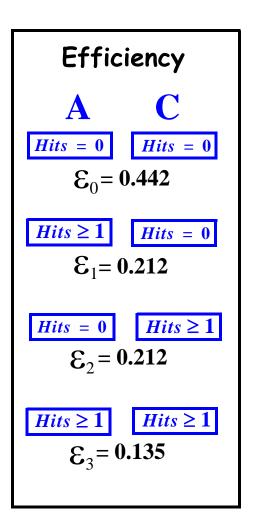
Moi	nte Carlo	Data
$\epsilon^{\text{sing}} =$	0.420	24664 / 82312 = 0.2996 +0.0016
ε ^A =	0.239	14146 / 82312 = 0.1719 +0.0013
ε ^C =	0.243	12738 / 82312 = 0.1548 +0.0013
E ^{coinc} =	0.0625	2220 / 82312 = 0.0270 + -0.0006 ($2733 / 101289 = 0.0270 + -0.0006$ for loser cuts)

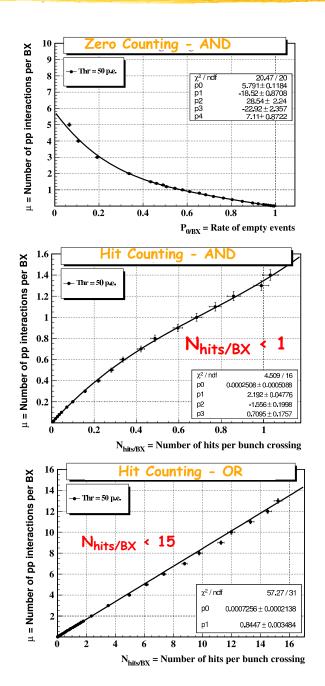
	\mathbf{M}	Ionte Carlo	Data
Ncoinc Nhits/pp	=	0.189	6653 / 86831 = 0.0766 +-0.0009
Nsing Nhits/pp	=	0.719	40765 / 86831 = 0.469 +0.002

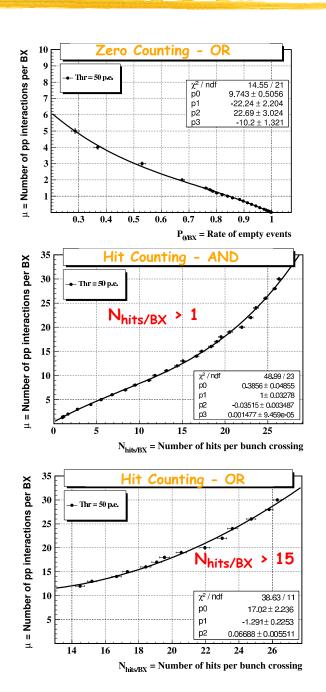


LUCID 14 TeV











LUCID



What needs to be done?

Fits for all possible LHC energies are needed.

Study of systematic errors.

Studies using real event samples at low μ .

Studies of the detector response using real event samples.



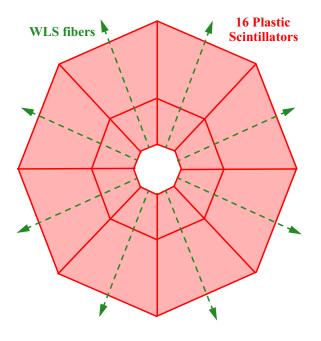
MBTS



MBTS has 16 plastic scintillators one each side. They are arranged in two rings and read-out by WLS fibers via the TileCal electronics. The signals are discriminated and provide a hit pattern that are used to do four measurements:

The efficiency of the detector is very high for non-diff. events:

$$\xi_0 = 0$$
 $\xi_1 = 0.004$ $\xi_2 = 0.004$ $\xi_3 = 0.992$ (14 TeV, only non-diff.)



For the two zero-counting-methods, MBTS uses the following function that is fitted to the event probability (P):

$$\mu = -p_0 - p_1 \ln(P)$$
 where $P = \frac{N_{00}}{N_{BX}}$ or $\frac{N_0}{N_{BX}}$ and p_0 , p_1 are constants

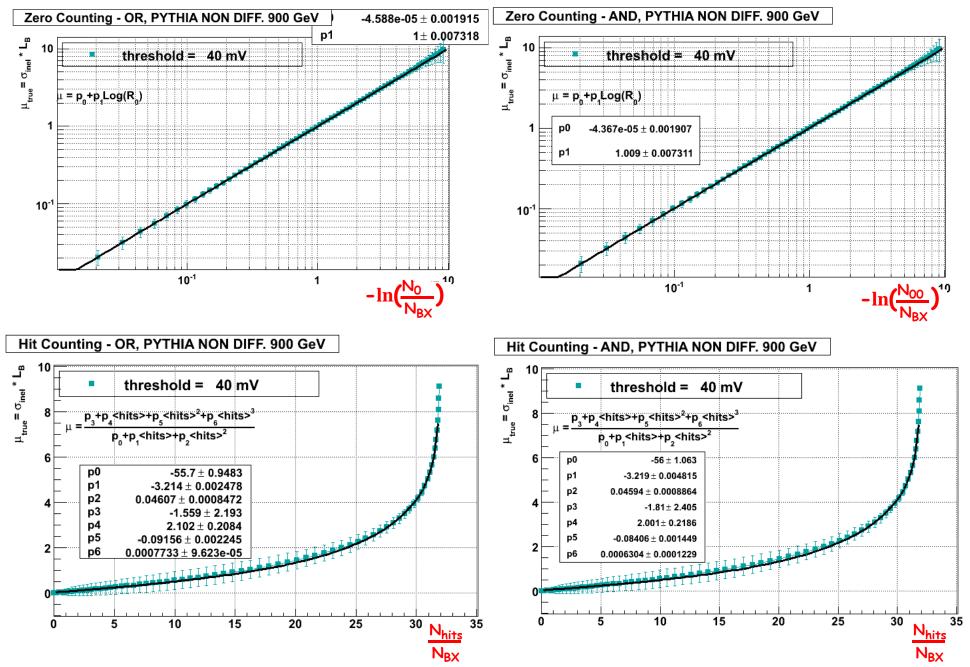
For hit counting, MBTS plot μ as a function of the hit probability (P) and fit a ratio of two polynomial functions so that μ can be expressed as

$$\mu = \frac{p_3 + p_4 P + p_5 P^2 + p_6 P^3}{p_0 + p_1 P + p_2 P^2}$$
 where $P = \frac{N_{hits}}{N_{BX}}$ and p_0, \dots, p_6 are constants



MBTS 900 GeV - Only non-diff.

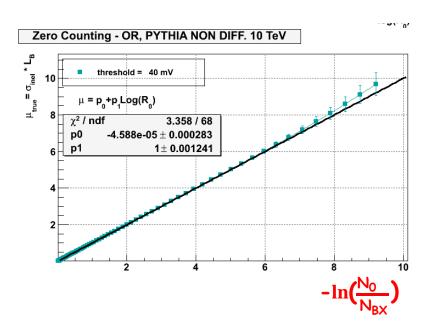


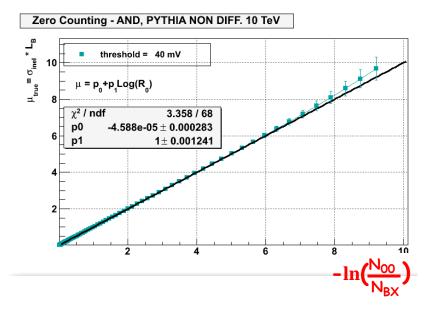


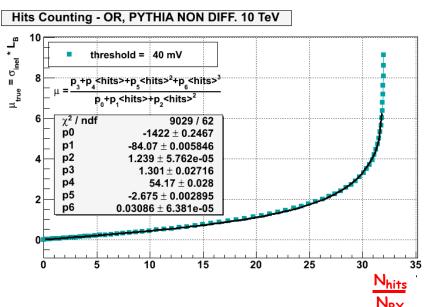


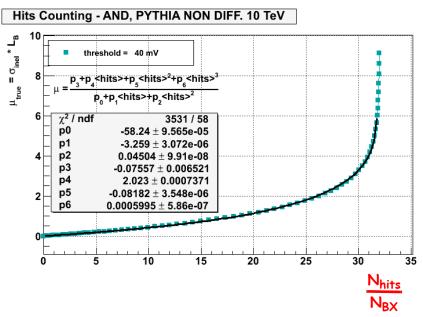
MBTS 10 TeV - Only non-diff.













MBTS



What needs to be done?

Monte Carlo samples with a mixture including diffractive events have to be used.

Fits for all possible LHC energies are needed.

Studies of the detector response using real event samples.

Studies where only a few of the 32 segments are used.

Correct for pulses that are longer than the bunch separation time.



Beam separation scans

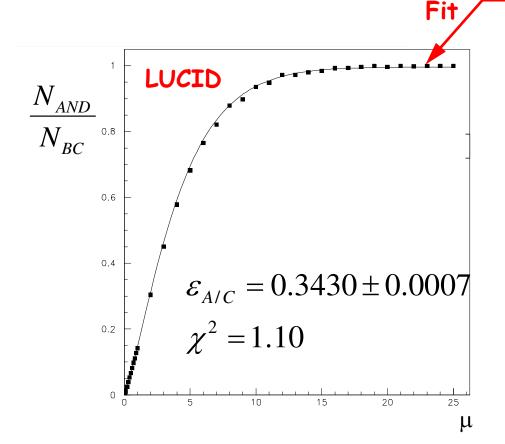


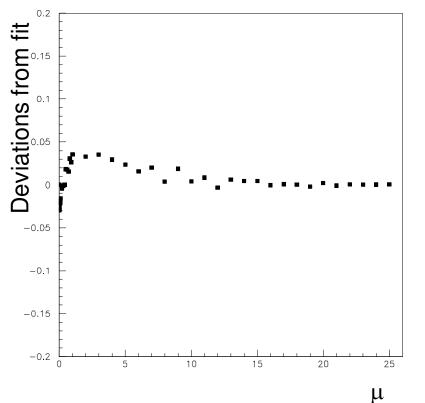
For Event-Counting-AND the probability function is _

$$N_{AND/BX} = 1 - e^{-\epsilon^A \mu} - e^{-\epsilon^C \mu} + e^{-(\epsilon^A + \epsilon^C - \epsilon^{Coin})\mu}$$

Assume that $\mathcal{E}^A = \mathcal{E}^C$ and that $\mathcal{E}^{coin} = \mathcal{E}^{A_x} \mathcal{E}^C$

 $N_{AND/BX} = 1-2e^{-\epsilon^{A}\mu} + e^{(2\epsilon^{A}-\epsilon^{A}\epsilon^{A})\mu}$





The fit works surprisingly well!

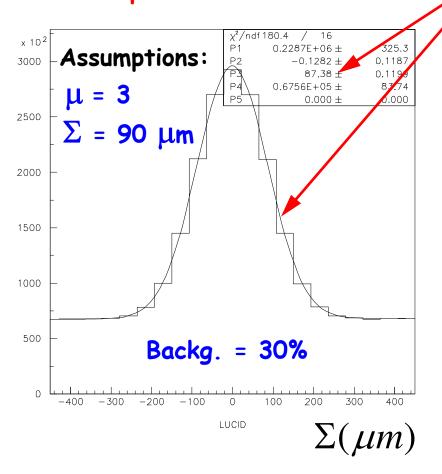


Beam separation scans



A separation scan done at $\mu>1$ will be distorted by the non-linear behaviour of the rate measurement.

Beam separation scan with LUCID



A Gaussian fit gives $\Sigma = 87.4 \mu m$

A fit with $N_{AND/BX} = 1 - 2e^{-\epsilon^{A}\mu} + e^{(2\epsilon^{A} - \epsilon^{A}\epsilon^{A})\mu}$ gives:

True Measured Difference μ : 3 3.2 8% Σ : 90 μ m 90.8 μ m 0.9% ϵ^{A} : 0.343 0.321 7%

The fit works better at higher μ where the distortions are larger !

A second fit with $N_{OR/BX} = 1 - e^{-\epsilon^{sing}\mu}$ to Event-Counting-OR events give ϵ^{sing} .



Final remarks



All detectors have done a lot of progress towards the goal of having realistic luminosity algorithms.

But we are not there yet!

Due to a lack of manpower the progress is slow (and LHC running does not help).

We will have a lot of work to do also in 2010!

LIST OF METHODS



Zero-counting-AND



A

C

Zero-counting-AND events are events with no signals in both detectors:

$$Hits = 0$$

Hits = 0

The probability to have a zero-AND event is

$$N_{00/BX} = e^{(\epsilon_0 - 1)\mu} = e^{-\epsilon^{sing}\mu} = \frac{N_{00}}{N_{BX}} = \frac{Number of measured zero-AND events}{Number of bunch crossings}$$

This function can easily be inverted so that μ can be obtained from the measured events:

$$\mu = \frac{\ln{\left(\frac{N_{00}}{N_{BX}}\right)}}{\epsilon_0 - 1} = \frac{\ln{\left(\frac{N_{00}}{N_{BX}}\right)}}{-\epsilon^{sing}}$$
 where only one parameter (\epsilon_0) has to be determined from Monte Carlo

The instantaneous luminosity is the obtained from

$$L = \frac{R_{in}}{\sigma_{in}} = \frac{f_{BX}}{\sigma_{in}} \times \frac{\ln{(\frac{N_{00}}{N_{BX}})}}{\epsilon_{0}-1}$$
 where σ_{in} = 79-83 mb at 14 TeV ϵ_{0} = 0.442 (LUCID) = 0.711 (BCM) = 0 (MBTS) all at 14 TeV

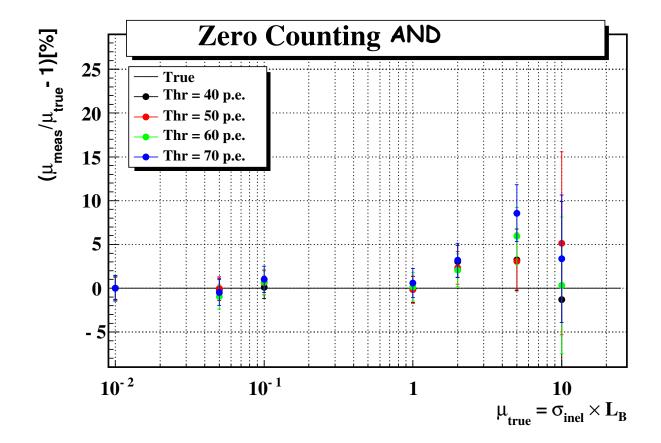


Zero-counting-AND



Even if one can find a simple expression of μ as a function of the number of measured empty events, this does not mean that this expression gives a perfect estimation of μ .

LUCID:



The reason for the error in the μ -determination above is migration i.e. the effect of particles with a low pulseheight (below the discriminator threshold) that add up at large μ and produce a total signal above threshold.



Event-counting-OR



Event-counting-OR events are events with a signal in at least one detector:

Probability(Zero-counting-AND) = 1 - Probability(Event-counting-OR)

A C $Hits \ge 1$ $Hits \ge 1$ Hits = 0 $Hits \ge 1$

The probability to have an event-counting-OR event is

$$N_{OR/BX} = 1 - e^{(\epsilon_0 - 1)\mu} = 1 - e^{-\epsilon^{sing}\mu} = \frac{N_{OR}}{N_{BX}} = \frac{N_{umber of measured event-counting-OR events}}{N_{umber of bunch crossings}}$$

$$N_{OR/BX} \approx \varepsilon_{Sing} \mu \text{ if } \mu \leftrightarrow 1$$

This function can easily be inverted so that $\boldsymbol{\mu}$ can be obtained from the measured events:

$$\mu = \frac{\ln(1 - \frac{N_{OR}}{N_{BX}})}{\varepsilon_{O} - 1} = \frac{\ln(1 - \frac{N_{OR}}{N_{BX}})}{-\varepsilon^{sing}}$$

where only one parameter (ϵ_0) has to be determined from Monte Carlo

The instantaneous luminosity is the obtained from

$$L = \frac{R_{in}}{\sigma_{in}} = \frac{f_{BX}}{\sigma_{in}} \times \frac{\ln(1 - \frac{N_{OR}}{N_{BX}})}{\varepsilon_{0} - 1}$$
 where $\sigma_{in} = 79 - 83$ mb at 14 TeV

where
$$\sigma_{\text{in}}$$
 = 79-83 mb at 14 TeV ϵ_0 = 0.442 (LUCID) = 0.711 (BCM) = 0 (MBTS) at 14 TeV



Zero-counting-OR



A

C

Hits = 0

Hits = 0

 $Hits \ge 1$

Hits = 0

Hits = 0

 $Hits \ge 1$

The probability to have a zero-AND event is given by

$$N_{O/BX} = \frac{N_0}{N_{BX}} = \frac{N_{umber of measured zero-OR events}}{N_{umber of bunch crossings}}$$

$$N_{O/BX} = e^{(\epsilon_0 + \epsilon_1 - 1)\mu} + e^{(\epsilon_0 + \epsilon_2 - 1)\mu} - e^{(\epsilon_0 - 1)\mu} = e^{-\epsilon^A \mu} + e^{-\epsilon^C \mu} - e^{-(\epsilon^A + \epsilon^C - \epsilon^{Coin})\mu} = \frac{N_0}{N_{BX}}$$

$$N_{O/BX} = 2e^{(\epsilon_0 + \epsilon_1 - 1)\mu} - e^{(\epsilon_0 - 1)\mu} = 2e^{-\epsilon^A \mu} - e^{(2\epsilon^A - \epsilon^{Coin})\mu} = \frac{N_0}{N_{BX}} \quad \text{if } \epsilon_1 = \epsilon_2 \text{ and } \epsilon^A = \epsilon^C$$

This function cannot be inverted analytically and it depends on two parameters (ϵ_0 and ϵ_1).



Event-counting-AND



A

 $Hits \geq 1$

 $Hits \geq 1$

Event-counting-AND events are events with a signals in both detectors:

Probability(Zero-counting-OR) = 1 - Probability(Event-counting-AND)

The probability to have an event-counting-AND event is given by

$$N_{AND/BX} = \frac{N_{AND}}{N_{BX}} = \frac{N_{umber of measured event-counting-AND events}}{N_{umber of bunch crossings}}$$



$$N_{AND/BX} = 1 - e^{(\epsilon_0 + \epsilon_1 - 1)\mu} - e^{(\epsilon_0 + \epsilon_2 - 1)\mu} + e^{(\epsilon_0 - 1)\mu} = 1 - e^{\epsilon^A \mu} - e^{\epsilon^C \mu} + e^{-(\epsilon^A + \epsilon^C - \epsilon^{Coin})\mu}$$

$$N_{AND/BX} = 1 - 2e^{(\epsilon_0 + \epsilon_1 - 1)\mu} + e^{(\epsilon_0 - 1)\mu} = 1 - 2e^{\epsilon^A \mu} + e^{(2\epsilon^A - \epsilon^{Coin})\mu} \text{ if } \epsilon_1 = \epsilon_2 \text{ and } \epsilon^A = \epsilon^C$$

This function cannot be inverted analytically and it depends on two parameters (ϵ_0 and ϵ_1).

$$N_{AND/BX} = \epsilon^{Coin} \mu$$
 if $\mu \ll 1$



Event-counting-XOR



 $A \\ Hits \ge 1$

Hits = 0

Event-counting-XOR events are events with signals in only one detector:

or

Hits = 0

 $Hits \ge 1$

The probability to have an event-counting-XOR event on side A is given by

$$N_{A/BX} = \frac{N_A}{N_{BX}} = \frac{N_{umber of measured event-counting-XOR-side A events}}{N_{umber of bunch crossings}}$$

$$N_{A/BX} = e^{(\epsilon_0 + \epsilon_1 - 1)\mu} - e^{(\epsilon_0 - 1)\mu}$$

and for side C one gets

$$N_{C/BX} = e^{(\epsilon_0 + \epsilon_2 - 1)\mu} - e^{(\epsilon_0 - 1)\mu}$$

These functions cannot be inverted analytically and they depends on two parameters.



Hit-counting-OR



Hit-counting-OR is a method which uses the hits in any detector:

A C

Hits
$$\geq 1$$
 Hits ≥ 1

Hits ≥ 0 Hits ≥ 1

If the detectors counted the true number of particles then

$$\mu = \frac{N_{part/BX}}{N_{part/pp}} = \frac{\text{The average number of detected particles per bunch crossing}}{\text{The average number of detected particles per inelastic pp interaction}}$$

Instead the detectors count hits and at large μ there will be several particles in each detector element. The number of particles can, however, be estimated from the number of hits:

$$N_{part} = -N_{tubes} \times \ln \left(1 - \frac{N_{hits}}{N_{tubes}}\right)$$
 where N_{tubes} is the number of detector elements i.e. tubes in the case of LUCID and scintillator segments in the case of MBTS.

and so

$$\mu = \frac{\ln \left(1 - \frac{N_{\text{hits/BX}}}{N_{\text{tubes}}}\right)}{\ln \left(1 - \frac{N_{\text{hits/pp}}}{N_{\text{tubes}}}\right)}$$



Hit-counting-AND



Hit-counting-AND is a method which uses the hits under the condition that both sides have hits:

$$\begin{array}{ccc} \mathbf{A} & \mathbf{C} \\ Hits \geq \mathbf{1} & Hits \end{array}$$

The average number of particles in a bunch crossing can be estimated in this mode from the expected number of hits for one inelastic pp interaction and the efficiencies. Assuming that the detector response on side A and side C are identical one gets

$$N_{\text{part/BX}} = 2\mu N_{\text{part/pp}}^{\text{A}} \left(1 - e^{-\mu \epsilon^{\text{A}}}\right) - \mu N_{\text{part/pp}}^{\text{Coinc}} \left(1 - 2e^{\mu \epsilon^{\text{A}}}\right) \qquad \text{if} \quad N_{\text{part/pp}}^{\text{A}} = N_{\text{part/pp}}^{\text{C}} \quad \text{and} \quad \epsilon^{\text{A}} = \epsilon^{\text{C}} = \epsilon^{\text{C}}$$

where N_{part} is obtained from N_{hits} by $N_{part} = -N_{tubes} \times \ln \left(1 - \frac{N_{hits}}{N_{tubes}}\right)$

$$N_{\text{hits/BX}} = N_{\text{tubes}} \left[1 - e^{-(2\mu N_{\text{part/pp}}^{A}(1 - e^{-\mu \epsilon^{A}}) - \mu N_{\text{part/pp}}^{\text{Coinc}}(1 - 2e^{-\mu \epsilon^{A}}))/N_{\text{tubes}}} \right]$$

What is needed is μ expressed as N_{hits} but the function above cannot be inverted analytically.

$$N_{\text{hits/BX}} = N_{\text{tubes}} \left[1 - e^{\mu (N_{\text{part/pp}}^{\text{Coinc}} - 2N_{\text{part/pp}}^{\text{A}})/N_{\text{tubes}}} \right]$$
 for large μ

$$N_{hits/BX} = \mu N_{part/pp}^{Coinc}$$
 for small μ



Statistics at low μ



Zero-AND

Event-OR

Compare the methods zero-counting-AND with event-counting-OR: Hits = 0

$$Hits = 0$$

$$Hits = 0$$

 $Hits \geq 1$

 $Hits \geq 1$

Zero-counting-AND:
$$e^{(\epsilon_0-1)\mu} = \frac{N_{00}}{N_{PV}}$$
 Event-counting-OR: $1 - e^{(\epsilon_0-1)\mu} = \frac{N_{OR}}{N_{BX}}$

1 -
$$e^{(\epsilon_0-1)\mu} = \frac{N_0}{N_0}$$

$$Hits \ge 1$$

$$Hits = 0$$

$$Hits = 0$$

$$Hits \ge 1$$

Conclusion:
$$N_{00} = N_{BX} - N_{OR}$$

where N_{BX} is a constant and so both methods have the same statistical uncertainty.

Event-AND

Event-OR

Compare the methods event-counting-AND with event-counting-OR: $Hits \ge 1$

$$Hits \geq 1$$

 $Hits \geq 1$

 $Hits \geq 1$

 $Hits \geq 1$

 $Hits \geq 1$

Hits = 0

Hits = 0

 $Hits \geq 1$

Compare the methods event-counting-OR with hit-counting-OR:

$$N_{OR/BX}$$
 = 1 - $e^{(\epsilon_0-1)\mu}$ $\approx \mu(1-\epsilon_0)$ = 0.56 μ for μ << 1 and 14 TeV

Nhits/BX = N_{tubes}
$$\left[1 - \left(1 - \frac{N_{\text{hits/pp}}}{N_{\text{tubes}}}\right)^{\mu}\right] \approx \mu N_{\text{hits/pp}} = 1.20 \mu \text{ for } \mu << 1 \text{ and } 14 \text{ TeV}$$

Conclusion: $N_{hits} = 2N_{OR}$

Conclusion: NOR > NAND